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Concept of Hydrodynamic Load Analysis of Fixed Jacket Structure - An Overview of Horizontal Cylinder

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Abstract : This paper focuses on the analysis of hydrodynamic loads on fixed offshore structures (horizontal cylinder) that are operating in shallow water and are often subjected to huge wave loading. For the purpose of this study, linear (Airy) wave theory was adopted together with the application of (21) in the load computation. The loads for six different sea states were computed using spread sheet for the following values of time t = 0, T/4, T/2.

Keywords: Airy wave theory, Fixed Jacket, Morrison equation, Horizontal cylinder,

1. Introduction

Hydrodynamic wave loading on fixed offshore structures has been an issue of concern to the offshore oil and gas industry. The analysis, design and construction of offshore structures is arguably one of the most demanding sets of tasks faced by the engineering profession. Over and above the usual conditions and situations met by land-based structures, offshore structures have the added complication of being placed in an ocean environment where hydrodynamic interaction effects and dynamic response become major considerations in their design. In general, wave and current can be found together in different forms in the ocean. The existence of waves and currents and their interaction play a significant role in most ocean dynamic processes and are important for ocean engineers (Haritos, 2007)

1.1 Hydrodynamics

Hydrodynamics is concerned with the study of water in motion. In the context of an offshore environment, the water of concern is the ocean.

Its motion, (the kinematics of the water particles) stems from a number of sources including slowly varying currents from the effects of the tides and from local thermal influences and oscillatory motion from wave activity that is normally wind-generated (Haritos, 2007).

2.0 Methodology

The jacket structure used for this study is a HD accommodation platform to be operated in shallow water and is similar to all fixed jacket offshore structures. The part of the structure under water was discretized in to (264) Beam elements. The water depth for the HD field is approximately 25.3m. The loads were computed using spread sheet.

Table 1: most probable wave heights and time periods for different sea states (Area 59) west Africa.

Sea State	Hs (m)	Tp (sec)	ζa (m)	t (sec)	k
1	1	5.5	0.5	4.125	0.133
2	2	5.5	1	4.125	0.133
3	3	6.5	1.5	4.875	0.0961
4	4	7	2	5.25	0.08371
5	5	7.5	2.5	5.625	0.0739
6	6	7.5	3	5.625	0.0739

2.1 Wave Theories

All wave theories obey some form of wave equation in which the dependent variable depends on physical phenomena and boundary conditions (Al-Salehy, 2002). In general, the wave equation and the boundary conditions may be linear and non linear.

2.2.1 Airy Wave Theory

The surface elevation of an Airy wave amplitude ζ_a, at any instance of time t and horizontal position x in the direction of travel of the wave, is denoted by η(x,t) and is given by:

$$\eta(x,t)=\zeta_a\cos(kx-\omega t) \qquad (1)$$

where wave number k = 2π/L in which L represents the wavelength (see Fig. 1) and circular frequency ω = 2π/T in which T represents the period of the wave. The celerity, or speed, of the wave C is given by L/T or ω/k, and the crest to trough wave height, H is given by $2\zeta_a$.

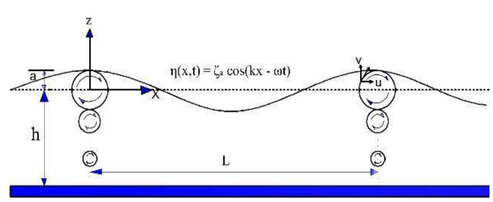

Figure 1: definition diagram for an Airy wave (Haritos, 2007).

the along wave u(x, t) and vertical v(x, t) water particle velocities in an Airy wave at position z measured from the Mean Water Level (MWL) in depth water h are given by:

$$u(x,t) = \frac{\omega\zeta_a \cosh[k(z+h)]}{\sinh(kh)}\cos(kx-\omega t) \quad (2)$$

$$v(x,t) = \frac{\omega\zeta_a \sinh[k(z+h)]}{\sinh(kh)}\sin(kx-\omega t) \quad (3)$$

The dispersion relationship relates wave number k to circular frequency ω (as these are not independent), via:

ω^2 = gktanh (kh) (4)

where g is the acceleration due to gravity (9.8 m/s^2). The along wave acceleration $\dot{u}$(x, t) is given by the time derivative of (2) as:

$$\dot{u}(x,t)=\frac{\omega^2\zeta_a \cosh[k(z+h)]}{\sinh(kh)}\sin(kx-\omega t) \quad (5a)$$

while the vertical velocity $\dot{v}$(x, t) is given by the time derivative of (3) as:

$$\dot{v}(x,t)=-\frac{\omega^2\zeta_a \sinh[k(z+h)]}{\sinh(kh)}\cos(kx-\omega t) \quad (5b)$$

It should be noted here that wave amplitude, $a=\zeta_a$, is considered small (in fact negligible) in comparison to water depth h in the derivation of Airy wave theory.
For deep water conditions, kh > π, (2) to (5) can be approximated to:
u(x,t)= $\omega\zeta_a e^{kz}\cos(kx-\omega t)$ (6) v(x,t)= $\omega\zeta_a e^{kz}\sin(kx-\omega t$ (7)

ω^2= gk (8)

$\dot{v}$(x,t)= $\omega^2\zeta_a e^{kz}\sin(kx-\omega t)$ (9)

This would imply that the elliptical orbits of the water particles associated with the general Airy wave description in (2) and. (3), would reduce to circular orbits in deep water conditions as implied by (6) and (7).

2.2.2 Stoke's Second Order Wave Theory

Stokes employed perturbation techniques to solve the wave boundary value problem and developed a theory for finite amplitude wave that he carried to the second order. In this theory, all the wave characteristics (velocity potential, celerity, surface profile, particle kinetics….e.tc) are formulated in terms of a power series in successively higher orders of the wave steepness (H/L).
A condition of this theory is that (H/d) should be small so that the theory is applicable only in deep water and a portion of the immediate depth range.
For engineering applications, the second-order and possibly the fifth-order theories are the most commonly used (Sorensen, 2006).
Stoke's wave expansion method is formally valid under the conditions (Iraninejad, 1988):

H/d << (kd)2 for kd < 1 and H/L << 1.

Stoke's wave theory is considered most nearly valid in water where the relative depth (D/L) is greater than about (1/10) (Patal, 1989).
Stoke's theory would be adequate for describing water waves at any depth of water. In shallow water, the connective terms become relatively large, the series convergence is slow and erratic and a large number of terms are required to achieve a uniform accuracy (Muga, 2003).
The fluid particle velocities are then given by:

$$v_x=\frac{\pi H}{T}\frac{\cosh[k(z+h)]}{\sinh(kh)}\cos(k(kx-\omega t))+$$
$$\frac{3(\pi H)^2}{4TL}\frac{\cosh[2k(z+h)]}{sinh^4(kh)}cos\ 2(kx\omega t) \quad (9a)$$

$$v_z=\frac{\pi H}{T}\frac{\sinh[k(z+h)]}{\sinh(kh)}\sin(k(kx-\omega t))+$$
$$\frac{3(\pi H)^2}{4TL}.\frac{\sinh[2k(z+h)]}{sinh^4(kh)}sin2(kx-\omega t) \quad (9b)$$

The fluid particle accelerations are then given by:

$$a_x=2\frac{\pi^2 H}{r^2}\frac{\cosh[k(z+h)]}{\sinh(kh)}\sin(kx-\omega t)+$$
$$\frac{3\pi^3 H^2}{T^2 L}\frac{\cosh[2k(z+h)]}{sinh^4(kh)}sin2(kx-\omega t) \quad (10a)$$

$$a_z=2\frac{\pi^2 H}{r^2}\frac{\sinh[k(z+h)]}{\sinh(kh)}cos(kx-\omega t)+$$
$$\frac{3\pi^3 H^2}{T^2 L}\frac{\sinh[2k(z+h)]}{sinh^4(kh)}cos2(kx-\omega t) \quad (10b)$$

These velocities and accelerations in (9) and (10) are used in Morison's equation to calculate load vectors of hydrodynamic loading by using Stoke's wave theory after being transformed from global coordinates for each member of the offshore structure.

2.2.3. Morison's Equation

The along wave or in-line force per unit length acting on the submerged section of a rigid vertical surface-piercing cylinder, F (z, t), from the interaction of the wave kinematics at position z from the MWL
(see Fig. 2), is given by Morison's quation.
This equation is originally developed to compute hydrodynamic forces acting on a cylinder at a right angle to the steady flow, and is given by:

$$F(z,t)=\rho\pi\frac{D^2}{4}C_m\, a(z,t) + \frac{1}{2}\rho D C_d\, V|V|(z,t) \quad (11)$$

in this equation, it is assumed that the wave force is acting on the vertical distance (z, t) of the cylinder due to the velocity (v) and acceleration (a) of the water particles, where (ρ) is the density of water, (D) is the cylinder diameter, (C_m) and (C_d) are inertia and drag coefficients, F_D and F_I are drag force and inertia force (Madhujit and Sinha, 1988).

$$F_D=\frac{1}{2}\rho D C_d\, V|V|\,(z,t) \quad (12)$$

$$F_I=\rho\pi\frac{D^2}{4}C_m\, a(z,t) \quad (13)$$

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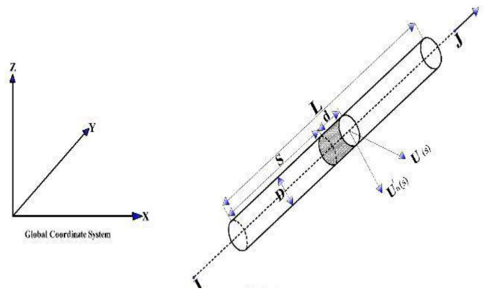
These coefficients are found to be dependent upon Reynold's number, *Re*, Keulegan Carpenter number, KC, and the β parameter, Viz;

$$KC = \frac{u_m}{D}T; \beta = \frac{Re}{KC} \qquad (14)$$



(i)

Figure 2: Water Particle Velocities along Member I [20]

Where u_m = the maximum along wave water particle velocity. It is found that for KC < 10, inertia forces progressively dominate; for 10 < KC < 20 both inertia and drag force components are significant and for KC > 20, drag force progressively dominates (Haritos, 2007).

The method adopted here assumes that only the components of water particles and accelerations normal to the member produce (Qian and Wang, 1992).

To formulate the hydrodynamic load vector F_w, consider the single, bottom mounted cylindrical member as shown in Fig. 2.

The forces are found by the well known semi-empirical Morison's formula (11). It also represents the load exerted on a vertical cylinder, assuming that the total force on an object in the wave is the sum of drag and inertia force components. This assumption (introduced by Morison) takes the drag term as a function of velocity and the inertia force as a function of acceleration (Zienkiewicz et al, 1978: Dean and Dalrymple, 1984: McCormick, 1973), so that:

$$F_n = \rho\,\pi\,\frac{D^2}{4}\,C_m\,v'_n - (C_m - 1)\,\rho\,\pi\,\frac{D^2}{4}\,u''_n$$

$$+\frac{1}{2}\rho D C_d(v_n - u'_n)\,|(v_n - u'_n)| \qquad (15a)$$

This can be simplified to:

$$F_n = \rho\,\pi\,\frac{D^2}{4} C_m\,v'_n$$

$$+\frac{1}{2}\rho\,D\,C_d\,(v_n)\,.\,|(v_n)| \qquad (15b)$$

Where:

F_n = nodal hydrodynamic force normal to the cylinder, D = Outer diameter of cylinder,

ρ = Sea water density. C_d = Drag coefficient (= 1.05). v'_n = water particle acceleration.

C_m = Inertia coefficient (= 1.2). v_n = water particle velocity. u'_n = Structural velocity. u''_n= Structural acceleration.

Equation (15b) neglects the non-linear terms of drag coefficient (Al-Jasim, 2000; Sarpakaya and Isaacson, 1981). water particle velocity and acceleration can be evaluated by potential velocity computed from wave theories; the absolute value of velocity is needed to preserve the sign variation of the force.

2.3 Global and Local System

The kinematics of cross-flow with resultant velocity (see Fig. 3) is;

$$[U_L] = [t][U_g] \qquad (16)$$

$$W_n = \sqrt{u' + w'} \qquad (17)$$

is determined using wave theory applied in the global system and then transferred to the local system using transformation matrix.

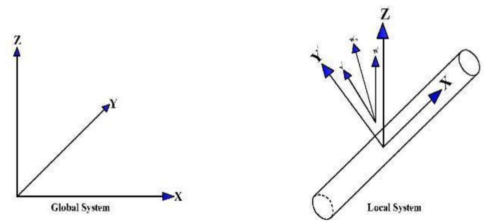


Figure 3: global and local system

Application of Morison's equation leads to:

$$F_L = \rho\,\pi\,\frac{D^2}{4}\,C_m\,\frac{du}{dt}$$

$$+\frac{1}{2}\rho\,D\,C_d\,(w_n)\,.\,|(w_n)| \qquad (18)$$

The components of the forces in the local axis system then become;

$$\begin{bmatrix} f'_y \\ f'_z \end{bmatrix} = \frac{1}{2}\rho\,D\,C_d\,(w_n)\,|(w_n)| \begin{bmatrix} u' \\ w' \end{bmatrix}$$

$$+\rho\,\pi\,\frac{D^2}{4}\,C_m \begin{bmatrix} u'' \\ w'' \end{bmatrix} \qquad (19)$$

To get the local forces, we need to get the member transformation matrix along the y - axis as follows;

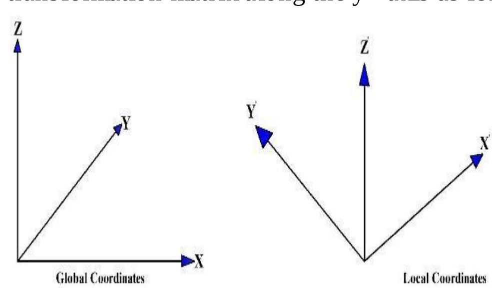


Figure 4: global and local coordinates for a horizontal cylinder (y-axis).

$$T = \begin{array}{c|ccc} & X & Y & Z \\ \hline x' & 0 & 1 & 0 \\ y' & -1 & 0 & 0 \\ z' & 0 & 0 & 1 \end{array}$$

$$T^{-1} = \begin{array}{c|ccc} & X & Y & Z \\ \hline x' & 0 & -1 & 0 \\ y' & 1 & 0 & 0 \\ z' & 0 & 0 & 1 \end{array}$$

And also the member transformation matrix along the x – axis is:

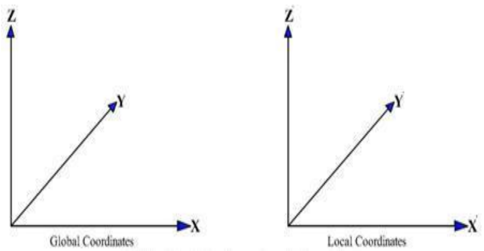


Figure 5: global and local coordinates for a horizontal cylinder (x-axis)

$$T = \begin{array}{c|ccc} & X & Y & Z \\ \hline x' & 1 & 0 & 0 \\ y' & 0 & 1 & 0 \\ z' & 0 & 0 & 1 \end{array}$$

$$T^{-1} = \begin{array}{c|ccc} & X & Y & Z \\ \hline x' & 1 & 0 & 0 \\ y' & 0 & 1 & 0 \\ z' & 0 & 0 & 1 \end{array}$$

Therefore, the local forces are given as:-

$$\begin{bmatrix} f'_x \\ f'_y \\ f'_z \end{bmatrix} = \frac{1}{2}\rho D C_d (w_n)\, |(w_n)| [T] \begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} + \rho \pi \frac{D^2}{4} C_m \begin{bmatrix} u'' \\ v'' \\ w'' \end{bmatrix} \qquad (20)$$

The local forces are then transferred in to global forces by the transpose matrix

$$F_w = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = [T]^{-1} \begin{bmatrix} 0 \\ f'_y \\ f'_z \end{bmatrix} \qquad (21)$$

3.0 Tables & Results

The following tables present the magnitude of hydrodynamic forces for three different sea states at different time phase. These forces serve as input loads for design.

Table 2: Hydrodynamic Loads @ time t = 0, distance x = 0

Load Component = x		**Load Type = Global**		
Sea States =		**1**	**2**	**3**
Hs (m) =		1	2	3
Tz (sec) =		5.5	5.5	6.5
Y (m)	**Z (m)**	**F_x (N/m)**	**F_x (N/m)**	**F_x (N/m)**
0	-26.8	-679.19	-677.71	-671.81
2.1	-26.8	-679.19	-677.71	-671.81
4.2	-26.8	-679.19	-677.71	-671.81
6.3	-26.8	-679.19	-677.71	-671.81
8.4	-26.8	-679.19	-677.71	-671.81
10.5	-26.8	-679.19	-677.71	-671.81
12.6	-26.8	-679.19	-677.71	-671.81
14.7	-26.8	-679.19	-677.71	-671.81
16.8	-26.8	-679.19	-677.71	-671.81
18.9	-26.8	-679.19	-677.71	-671.81
21.0	-26.8	-679.19	-677.71	-671.81

Table 2: Hydrodynamic Loads @ time t = T/4 and x = 16m for (3) sea states

Load Component = z		**Load Type = Global**		
Sea States =		**1**	**2**	**3**
Hs (m) =		1	2	3
Tz (sec) =		5.5	5.5	6.5
Y (m)	**Z (m)**	**F_x (N/m)**	**F_x (N/m)**	**F_x (N/m)**
0	-26.8	1.63	3.3	9.09
2.1	-26.8	1.63	3.3	9.09
4.2	-26.8	1.63	3.3	9.09
6.3	-26.8	1.63	3.3	9.09
8.4	-26.8	1.63	3.3	9.09
10.5	-26.8	1.63	3.3	9.09
12.6	-26.8	1.63	3.3	9.09
14.7	-26.8	1.63	3.3	9.09
16.8	-26.8	1.63	3.3	9.09
18.9	-26.8	1.63	3.3	9.09
21	-26.8	1.63	3.3	9.09

5.0 Conclusion

It can be concluded that:

The magnitude of all hydrodynamic forces along the member is uniform at constant depth. Also, the magnitude of the hydrodynamic force in the *x*-component increases in a regular pattern with change in sea state while the magnitude of the hydrodynamic force in the z-component increases in an irregular pattern with change in sea states.

5.0 Acknowledgement

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